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A New Construction of Group and Nongroup Perfect Codes

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From two perfect 1-codes C' and C'' in cartesian products S' respective S'' we shall construct a perfect 1-code C in a cartesian product S. We shall show how the codes C' and C'' might be chosen so that the code C will be equivalent respective not equivalent to a subgroup of S. We shall also give an example of a perfect 1-code that is not equivalent to any "Vasilev" code.

1. INTRODUCTION AND NOTATIONS

Let $S = S_1 \times S_2 \times \cdots \times S_n$ be the cartesian product of the finite additive groups $S_1, S_2, ..., S_n$ and let $d(s, s')$ denote the distance between the elements s and s' of S, i.e.,

$$d(s, s') = |\{i \mid s_i \neq s_i' \; s = (s_1, ..., s_n) \; s' = (s_1', ..., s_n')\}|.$$

($|A|$ denotes the number of elements in a set A.) A subset C of S is called a perfect e-code if for any element s of S there is one and only one element c of C satisfying $d(s, c) \leqslant e$. If the number of elements in the sets $S_1, S_2, ..., S_n$ are not equal then a perfect code might be called a mixed perfect code.

We shall call a map π from S to S an equivalence of S if $\pi((s_1, ..., s_n)) = (\pi_1(s_{i_1}), ..., \pi_n(s_{i_n}))$ where $(i_1, ..., i_n)$ is a permutation of $(1, 2, ..., n)$ satisfying $|S_{i_k}| = |S_k|$ for $k = 1, 2, ..., n$ and $\pi_1, ..., \pi_n$ are permutation of $S_1, ..., S_n$ respective. Two subsets D and D' of S are said to be equivalent if $\pi(D) = D'$ where π is an equivalence of S.

In this paper, we shall from two given perfect 1-codes C' and C'' in cartesian products S' respective S'', construct a perfect 1-code C in a cartesian product S. We shall show how the codes C' and C'' might be choosen so that the code C will be equivalent respective not equivalent to a subgroup of S. Perfect 1-codes not equivalent to subgroups of S are constructed in Vasilev (1962). Vasilev's construction has been generalized in Schönheim (1968) and Lindström (1969). But we shall give an example of a perfect 1-code that is not equivalent to any perfect 1-code constructed as in Vasilev (1962). We shall also discuss an example in Herzog and Schönheim (1971) concerning nongroup mixed perfect 1-codes and give a less trivial example of a mixed perfect 1-code that is not equivalent to any subgroup of S.

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Since we only consider perfect 1-codes we shall often omit the prefix 1- in perfect 1-codes.

We need some further notations and definitions.

By the sum $A + B$ of two subsets A and B of an additive group S we shall mean the set of elements $a + b$ where $a \in A$ and $b \in B$.

A period of a subset A of an additive group S is an element g of S satisfying

$$g + A = A.$$

If I is any ordered set, $I = \{i_1, ..., i_n\}$ where $i_1 < \cdots < i_n$ and if there are sets $S_{i_1}, ..., S_{i_n}$ then by $S(I)$ we shall mean the cartesian product

$$S(I) = S_{i_1} \times \cdots \times S_{i_n}.$$

If I' is a subset of I and c an element of $S(I)$ then

$$c_{I'} = (c_{i_1'}, ..., c_{i_k'}) \qquad \text{where} \quad I' = \{i_1', ..., i_k'\} \text{ and } i_1' < \cdots < i_k'.$$

If $i \in I$ then we shall write c_i instead of $c_{\{i\}}$.

Let I' be a subset of I. ι will always mean the map from $S(I')$ to $S(I)$ defined by

$$\begin{aligned} (\iota(c))_i &= c_i \qquad \text{if} \quad i \in I', \\ &= 0 \qquad \text{if} \quad i \in I \backslash I'. \end{aligned}$$

If $s, s' \in I$ and I' is a subset of I then by the distance in I' between s and s' we shall mean

$$d_{I'}(s, s') = |\{i \in I' \mid s_i \neq s_i'\}|.$$

The weight in I' of an element s of $S(I)$ is defined by

$$w_{I'}(s) = d_{I'}(s, \bar{0}) \qquad \text{where } \bar{0} = (0, ..., 0).$$

The set

$$S_I(\bar{0}, e) = \{s \in S(I) \mid w(s) \leqslant e\}$$

will be called a sphere of radius e and center $\bar{0}$.

Finally, by $G^r(p)$ we shall mean the elementary abelian group of order p^r and type $(p, ..., p)$ for a prime p.

2. Two Theorems

Consider the perfect code C' in $S' = G^2(2) \times G(2) \times G(2) \times G(2) \times G(2)$ consisting of the elements

$$(0, 0, 0, 0, 0), (\alpha, 1, 0, 1, 0), (\beta, 1, 1, 0, 0), (1, 1, 0, 0, 1),$$
$$(0, 1, 1, 1, 1), (\alpha, 0, 1, 0, 1), (\beta, 0, 0, 1, 1), (1, 0, 1, 1, 0),$$

($G^2(2) = \{0, \alpha, \beta, 1\}$) and the perfect code C'' in $S'' = G(2) \times G(2) \times G(2)$ consisting of the elements $(0, 0, 0)$ and $(1, 1, 1)$. Let v denote any element of $G(2) \times G(2) \times G(2) \times G(2)$ and let p be the map from S' to $S = G(2) \times G(2) \times G(2) \times G(2) \times G(2) \times G(2) \times G(2)$ defined by

$$p(0, v) = (0, 0, 0, v) \qquad p(\alpha, v) = (1, 0, 0, v)$$
$$p(\beta, v) = (0, 1, 0, v) \qquad p(1, v) = (0, 0, 1, v)$$

and let ι denote the map from S'' to S defined by

$$\iota(u) = (u, \bar{0}) \qquad \text{if} \quad u \in S'' \quad \text{and} \quad \bar{0} = (0, 0, 0, 0).$$

It is easy to verify that the subset

$$C = p(C') + \iota(C'')$$

of S is a perfect 1-code in S. Actually it is possible to prove the following theorem.

THEOREM 1. *Suppose that $I_1 \cap I_2 = \varnothing$, that the sets S_i $i \in I_1 \cup I_2$ are additive groups and that $| S_{i_0} | = | S_{I_2}(0, 1)|$ where i_0 is an element of I_1. Let p be an injective map from $S(I_1)$ to $S(I)$ where $I = I_2 \cup I_1 \backslash \{i_0\}$ satisfying*

$$p(c)_{I \backslash I_2} = c_{I_1 \backslash \{i_0\}} \qquad \text{if} \quad c \in S(I_1) \text{ and } p(\iota(c)) \in S_{I_2}(0, 1) \text{ if } C \in S_{i_0}. \tag{1}$$

If C' and C'' are perfect 1-codes in $S(I_1)$ resp. $S(I_2)$ then

$$C = p(C') + \iota(C'')$$

is a perfect 1-code in $S(I)$.

Proof. Suppose that C' and C'' are perfect codes. Let c and c' be any elements of C', d and d' be any elements of C'' and let

$$k = d(p(c) + \iota(d), p(c') + \iota(d')).$$

We shall prove that $k = 0$ or $k \geqslant 3$. Consider the following four cases.

Case 1. If $c = c'$ and $d = d'$, then trivially $k = 0$.

Case 2. If $c = c'$ and $d \neq d'$, then, since C'' is a perfect code, $k \geqslant 3$.

Case 3. If $c \neq c'$ and $d = d'$, then from (1) and since C' is a perfect code we deduce that $k \geqslant 3$.

Case 4. If $c \neq c'$ and $d \neq d'$, then $d_{I \backslash I_2}(p(c), p(c')) \geqslant 2$, $d_{I_2}(p(c), p(c')) \leqslant 2$, and $d_{I_2}(d, d') \geqslant 3$. Consequently $k \geqslant 2 + (3 - 2) = 3$.

Note that we have proved that $k = 0$ if and only if $c = c'$ and $d = d'$. Consequently

$$|C| = |C'|\,|C''|. \tag{2}$$

It is an easy and well-known consequence of the definition of perfect e-code that

$$\text{If } D \text{ is a perfect } e\text{-code in } S(I), \text{ then } |D|\,|S_I(\bar{0}, e)| = |S(I)|. \tag{3}$$

Now, by (2), (3) and the equalities $|S_{I_1}(0, 1)| = |S_I(0, 1)|$ and $|S_{I_2}(0, 1)| = |S_{i_0}|$,

$$|C|\,|S_I(0, 1)| = |C'|\,|C''|\,|S_{I_1}(0, 1)| = |S(I_1)|\,|S(I_2)|/|S_{I_2}(0, 1)| = |S(I)|.$$

Since C obviously satisfies the necessary condition (3) for being a perfect code and since the minimum distance between any two elements of C is three it is now easy to see that C is a perfect code.

Unfortunately it is not so easy to use Theorem 1 to construct perfect codes in cartesian products $S(I)$ where the numbers $|S_i|\, i \in I$ are not powers of the same prime. Then as a consequence of that the numbers $S_i \iota \in I_1 \cup I_2$ could not be powers of a prime and perfect codes in such cartesian products are not known by the author.

Note that the construction of perfect codes given in Theorem 1 might be seen as a generalization of the construction given by a combination of Theorem 1 and the proof of Lemma 4 in Herzog and Schönheim (1972). But in our construction we can, if p is given, choose any perfect codes C' and C'' in $S(I_1)$ respective $S(I_2)$, C will always be a perfect code in $S(I)$. Of course some of the codes might not be equivalent. The following theorem gives, when C' is a subgroup of $S(I_1)$, under certain conditions on the sets S_i, a necessary and sufficiant condition on C'' so that C should be equivalent to a subgroup of $S(I)$.

THEOREM 2. *Suppose in addition to the assumptions in Theorem* 1 *that* $|I_1| > 1$, $S_i \subset S_{i_0}$ *for* $i \in I_2$, $S_{i_0} = \bigcup_{i \in I_2} S_i$, $S_i \cap S_j = \{0\}$ *if* $i \neq j$ $i, j \in I_2$ *and that*

$$\begin{aligned} (p(s_{i_0}))_i &= s_{i_0} \quad &\text{if } s_{i_0} \in S_i, \\ &= 0 \quad &\text{if } s_{i_0} \notin S_i. \end{aligned}$$

Let h be the map from $S(I_2)$ to S_{i_0} defined by

$$h(s) = \sum_{i \in I_2} s_i.$$

If C' is a perfect 1*-code and a subgroup of $S(I_1)$, C'' a perfect* 1*-code in $S(I_2)$, then the perfect* 1*-code*

$$C = p(C') + \iota(C'')$$

is equivalent to a subgroup of $S(I)$ *if and only if*

$$C'' = s + \ker h \tag{4}$$

where $s \in S(I_2)$ *and* $\ker h$ *is the kernel of* h.

Note that in Herzog and Schönheim (1971) it was proved that $\ker h$ always is a perfect code. We need two lemmas in the proof of Theorem 2.

LEMMA 1. *Suppose in addition to the assumptions in Theorem* 1 *that* $\bar{0} \in C'$ *and* $\bar{0} \in C''$. *Let* s *be an element of* $S(I)$.

(i) *If* $w_{I_2}(s) = w_{I \setminus I_2}(s) = 1$, *then* $d(p(c), s) = 1$ *for an element* c *of* C'.

(ii) *If* $s \in C$ *and* $w_{I_2}(s) \in 1$, *then* $s = p(c)$ *for an element* c *of* C'.

Proof. We shall only prove (i). The proof of (ii) is similar. Since $w_{I_2}(s) = 1$ we deduce that $w(s') = 2$ and $p(s') = s$ for an element s' of $S(I_1)$. Since C' is a perfect code and since $\bar{0} \in C'$, there is $c \in C'$, $w(c) = 3$, $d_{I_1}(c, s') = 1$ and $c_{i_0} = s'_{i_0}$. Since p satisfies (1) we deduce that $d(p(c), s) = 1$.

LEMMA 2. *Suppose that* D *is a subset of* $S(I)$ *where the sets* S_i $i \in I$ *are additive groups and that* D *contains the element* $\bar{0} = (0, \ldots, 0)$. *If* D *is equivalent to a subgroup* D' *of* $S(I)$, *then* $\pi(D) = D'$ *for an equivalence* π *of* $S(I)$ *satisfying* $\pi(\bar{0}) = \bar{0}$.

Proof. Suppose that π' is an equivalence of $S(I)$ and that $\pi'(D)$ is a subgroup of $S(I)$. Let π denote the permutation of $S(I)$ defined by

$$\pi(s) = \pi'(s) - \pi'(0) \qquad s \in S(I).$$

Then $\pi(D) = \pi'(D)$ since $\pi'(D)$ is a subgroup of $S(I)$ and $\bar{0} \in D$. Addition of a given element in $S(I)$ is always an equivalence, consequently π is an equivalence and the lemma is proved.

Proof of Theorem 2. We first show that if $C'' = \ker h$, then C is a subgroup of $S(I)$. Let c and c' be any two elements of C'. It follows from the definition of p and h that

$$p(c_{i_0} + c'_{i_0}) - p(c_{i_0}) - p(c'_{i_0}) \in \ker h.$$

Consequently

$$p(c) + p(c') - p(c + c') \in \iota(C'').$$

It is now easy to verify, since C' and C'' are subgroups of $S(I_1)$ respective $S(I_2)$, that C is a subgroup of $S(I)$.

Now suppose that C'' satisfies (4). Addition of $-\iota(s)$ to every element of $S(I)$ gives an equivalence π of $S(I)$. But, since $w_{I\setminus I_2}(s) = 0$,

$$\pi(p(C') - \iota(C'')) = p(C') + \ker h.$$

Consequently, $p(C') + \iota(C'')$ is equivalent to a subgroup of $S(I)$.

Now we shall show that if C is equivalent to a subgroup of $S(I)$, then (4) is true.

First consider the special case when $\bar{0} \in C''$. Suppose that C is equivalent to a subgroup of $S(I)$ and that $C'' = \ker h$. It follows from Lemma 2 that there is an equivalence π of $S(I)$, $\pi(C)$ is a subgroup of $S(I)$ and

$$(\pi(s))_i = \pi_i(s_i) \qquad \text{and} \qquad \pi_i(0) = 0 \quad \text{for } i \in I \text{ and } s \in S(I) \tag{5}$$

where π_i are permutations of S_i for $i \in I$. Let c be any element of $C''\setminus\ker h$. Since $\ker h$ is a perfect code, $d(c, c') = 1$ for an element c' of $\ker h$. Suppose that $c = \sum_{j \in J} \iota(c_j)$, $c' = \sum_{j \in J'} \iota(c_j')$ where J and J' are subsets of I_2, (and the elements c_j and c_j' are nonzero elements of S_j for $j \in J$ respective $j \in J'$. There are three possible cases.

Case 1. $J = J'$, $c_j = c_j'$ for $j \in J\setminus\{j_0\}$ and $c_{j_0} \neq c_{j_0}$ where $j_0 \in J$.

Case 2. $J \subset J'$, $|J| + 1 = |J'|$, and $c_j = c_j'$ if $j \in J$.

Case 3. $J \supset J'$, $|J'| + 1 = |J|$, and $c_j = c_j'$ if $j \in J'$.

We shall show that Case 2 contradicts the assumptions on C and C''. With the same argument it is possible to show that Cases 1 and 3 also gives a contradiction.

Let a be a nonzero element of S_{i_1} where $i_1 \in I\setminus I_2$ (exists since $|I_1| > 1$). By Lemma 1 there is for any $\nu \in J'$

$$a^{(\nu)} = \iota(a) + \iota(b_{i_\nu}) + \iota(c_\nu') \in p(C')$$

where $i_\nu \in I\setminus(I_2 \cup \{i_1\})$ and b_{i_ν} is a nonzero element of S_{i_ν}. Since the distance between any two elements of C' is greater then or equal to three and since $p^{-1}(a^{(\nu)}) \in C'$

$$i_\nu = i_{\nu'} \qquad \text{if} \quad \nu \neq \nu'. \tag{6}$$

C' is a subgroup of $S(I_1)$, consequently $p(\sum_{\nu \in J'} p^{-1}(a^{(\nu)}))$ belongs to the set $p(C')$ that is

$$|J'|\,\iota(a) + \sum_{\nu \in J'} \iota(b_{i_\nu}) + p\left(\sum_{\nu \in J'} c_\nu'\right) \in p(C').$$

But, since c' is an element of $\ker h$, $\sum_{\nu \in J'} c_\nu' = 0$. Consequently, since $\bar{0} \in C''$,

$$a' = |J'|\,\iota(a) + \sum_{\nu \in J'} \iota(b_{i_\nu}) \in C. \tag{7}$$

Now, since $\pi(C)$ is a subgroup of $S(I)$,

$$\sum_{\nu \in J'} \pi(a^{(\nu)}) - \pi(a') - \pi(c) \in \pi(C). \tag{8}$$

Let u be the element of $J' \backslash J$. It follows from (5), (6), (7), and (8) that

$$a'' = |J'| \, \pi(\iota(a)) - \pi(|J'| \, \iota(a)) + \pi(\iota(c_u')) \in \pi(C).$$

But since $w(a'') \leqslant 2$ and since $\bar{0} \in \pi(C)$, we conclude that $a'' = 0$. Consequently $c_u' = 0$, which is a contradiction.

Now to the general case. Suppose that C is equivalent to a subgroup of $S(I)$. Let c be any element of C''. If we add $-\iota(c)$ to every element of $S(I)$, then we get an equivalence π of $S(I)$. But, since $w_{I \backslash I_2}(\iota(c)) = 0$,

$$\pi(C) = p(C') + \iota(C'' - c).$$

Since the set of equivalences is a group, $\pi(C)$ is equivalent to a subgroup of $S(I)$. But, since $\bar{0} \in C'' - c$, it now follows from what we already have proved that $C'' - c = \ker h$ and the proof of Theorem 2 is complete.

3. Two Examples

It follows from Theorem 3 in Lindström (1975) that if some of the groups S_i $i \in I$ are not isomorphic to $G^r(p)$ for a prime p, then a perfect code never can be a subgroup of $S(I)$. Consequently, it is possible to construct perfect codes not equivalent to subgroups of $S(I)$ simply by substituting one of the groups S_ι with a group S_ι' of different group structure but with the same number of elements. This method was used in Example 2 of Herzog and Schönheim (1971). By Theorem 2 in this paper we do not have to use that trick to construct perfect codes not equivalent to subgroups as shown in the following example.

Example 1. Let $I_2 = \{1, 2, \ldots, 7\}$ and $S_i \cong G(2)$ for $i \in I_2$. Consider the subgroup C'' of $S(I_2)$ generated by

$$(1, 1, 1, 0, 0, 0, 0), \quad (1, 0, 0, 1, 1, 0, 0),$$
$$(1, 0, 0, 0, 0, 1, 1), \quad (0, 1, 0, 1, 0, 1, 0).$$

It is easy to see that C'' is a perfect code. We shall also consider the subgroup $\pi(C'')$ of $S(I_2)$ generated by

$$(0, 1, 1, 1, 0, 0, 0), \quad (0, 1, 0, 0, 1, 1, 0),$$
$$(1, 1, 0, 0, 0, 0, 1), \quad (0, 0, 1, 0, 1, 0, 1),$$

obtained from C'' by a permutation π of I_2.

By Theorem 2 in Lindström (1975) there is a group S_{i_0}, $S_i \subset S_{i_0}$ if $i \in I_2$, $S_{i_0} = \bigcup_{i \in I_2} S_i$, $S_i \cap S_j = \{0\}$ if $i \neq j$ and a map h from $S(I_2)$ to S_{i_0}, $h(s) = \sum_{i \in I_2} s_i$ and $C'' = \ker h$. By Lemma 1 in Herzog and Schönheim (1972), $S_{i_0} \cong G^3(2)$.

Assume that $i_0 \in I_1$, $S_i \cong G^3(2)$ if $i \in I_1$ and that $| I_1 | = 9$. Let C denote the Hamming code in $S(I_1)$, cf. van Lint (1971, p. 22). Consider the subset

$$C = p(C') + \iota(\pi(C'')) \qquad (p \text{ as in Theorem 2})$$

of $S(I_2 \cup I_1 \backslash \{i_0\})$. It is easy to control that there is no $s \in S(I_2)$ satisfying $\pi(C'') = s + C''$. Consequently, by Theorem 2, C is not equivalent to a subgroup of $S(I_2 \cup I_1 \backslash \{i_0\})$.

Now we shall compare the construction of perfect codes given by Theorem 1 and the construction of perfect codes in Vasilev (1962). First we give Vasilev's construction.

Suppose that $S_i \cong G(2)$ for $i \in \{1\} \cup I \cup I'$ where $I = \{2, \ldots, n\}$ and $I' = \{n + 1, \ldots, 2n - 1\}$. Let C be a perfect code in $S(I)$ and τ any map from $S(I)$ to S_1. Let σ be the map $\sigma(s) = \sum_{i \in I'} s_i$ from $S(I')$ to S_1. Then the set

$$C_\tau = \{(\tau(c) + \sigma(s), c + s, s) \mid s \in S(I'), c \in C\}$$

is a perfect code. It is easy to see that by a suitable choice of τ, C_τ will not be equivalent to a subgroup of $S(I \cup I' \cup \{1\})$.

PROPOSITION 1. *Let C_τ be defined as above. Suppose that $0 \in C$ and that $\tau(0) = 0$. If $c \in C_\tau$, $c_1 = 1$, and $w(c) = 3$, then c is a period of C_τ.*

Proof. If c is any element of C_τ of weight 3 and with $c_1 = 1$, then $w_I(c) = w_{I'}(c) = 1$. It easily follows from Vasilev's construction that any such element must be a period.

PROPOSITION 2 (notations from Theorem 2). *Suppose that π is an equivalence of $S(I_2)$ satisfying*

$$\text{if} \quad c \in \pi(\ker h) \cap \ker h \qquad \text{then} \quad w(c) \neq 3. \tag{9}$$

If i is any element of I then there is $c \in p(C') + \iota(\pi(\ker h))$, $w(c) = 3$, $c_i = 0$, and c is not a period of $p(C') + \iota(\pi(\ker h))$.

Proof. We only consider the case when $i \in I_2$. If $i \in I \backslash I_2$ the proof is similar. So suppose that $i \in I_2$. Let C denote the code $p(C') + \iota(\ker h)$ and C^π denote the code $p(C') + \iota(\pi(\ker h))$. By (9), since $\ker h$ and $\pi(\ker h)$ are perfect codes, there exists $j \in I_2$ and elements c and c^π of $\ker h$ respective $\pi(\ker h)$ satisfying

$$w(c) = w(c^\pi) = 3, \qquad c_i = c_j = c_i{}^\pi = c_j{}^\pi = -1, \qquad \text{and} \qquad c \neq c^\pi.$$

By Lemma 1, there exists elements $c^{(i)}$ and $c^{(j)}$ of C', $(p(c^{(\nu)}))_\nu = 1$ if $\nu = i$ or j and $w(c^{(i)}) = w(c^{(j)}) = 3$. But, since C is a subgroup of $S(I)$,

$$s = p(c^{(i)}) + p(c^{(j)}) + c \in C.$$

Now, since $w_{I_2}(s) = 1$, it follows from Lemma 1 that $s = p(c'')$ for an element c'' of C'. Consequently s also belongs to C^π. If $p(c^{(i)})$ should be a period of C^π then

$$s^\pi = p(c^{(i)}) + (p(c^{(j)}) + \iota(c^\pi)) \in C^\pi.$$

But this is impossible since $d(s, s^\pi) = 2$. The proposition is consequently proved, since $p(c^{(i)})$ is not a period of C^π.

These two propositions will make it easy for us to construct perfect codes not equivalent to any "Vasilev" code.

EXAMPLE 2. Let C'' and $\pi(C'')$ be as in Example 1. Let C' be a perfect code and a subgroup of $S(I_1)$ where $|I_1| = 9$, $i_0 \in I_1$, $S_{i_0} \cong G^3(2)$, $S_i \cong G(2)$ for $i \in I_1 \backslash \{i_0\}$ and $I_1 \cap I_2 = \varnothing$. For the existence of such a code see e.g. Herzog and Schönheim (1972). Consider the code

$$C^\pi = p(C') + \iota(\pi(C'')).$$

As in Example 1, $C'' = \ker h$ for a suitable map h from S_{i_0} to $S(I_2)$. It is easy to verify, for instance by inspection, that $\pi(C'') \cap C'' = \{\bar{0}\}$.

Now consider the group of periods G^π and G_τ for C^π and a "Vasilev" code C_τ in $S(I)$ where $I = I_2 \cup I_1 \backslash \{i_0\}$, $\tau(\bar{0}) = 0$, and $\bar{0} \in C$. Suppose that $\pi'(C^\pi) = C_\tau$ for an equivalence π' of $S(I)$. Since $S_i \cong G(2)$ for $i \in I$ we easily deduce that $\pi' = \pi_0 \circ \pi_1$ where π_0 and π_1 are equivalences of $S(I)$, π_0 is a permutation of I and π_1 is addition of an element s of $S(I)$. If $g \in G$ then g is a period of $s + C^\pi$. Also if g is a period of a set D then $\pi_0(g)$ is a period of $\pi_0(D)$. Consequently, since $\pi'^{-1} = \pi_1^{-1} \circ \pi_0^{-1}$, $\pi_0(G^\pi) = G_\tau$.

By Proposition 1 and since π_0 is a permutation of I, $\{c \in C^\pi \mid c_i = 1\ w(c) = 3\} \subset G^\pi$ for at least one element i of I, which contradicts Proposition 2.

As any "Vasilev" code is equivalent with a C_τ where $\tau(\bar{0}) = 0$ and $\bar{0} \in C$, we conclude, since the set of equivalences is a group, that C^π is not equivalent to any "Vasilev" code.

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